SPLiCE: Single-Point LiDAR and Camera Calibration & Estimation Leveraging Manhattan World (Supplementary Material)

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***Abstract*— In this supplementary material, we introduce a geometric approach for computing the LiDAR incident angle α, which compensates for possible misalignment between the LiDAR frame and the calibration board. This misalignment can introduce errors in the measurement, so the incident angle α helps to correct for this and improve the accuracy of the data captured by the LiDAR. The rotation angle θ of the calibration board is calculated from the tracking of the MW frame in the image data. The relationship between α and θ allows us to determine α, improving calibration accuracy. Furthermore, the accuracy of the MW frame tracking method is validated through a comparison with the ground truth obtained from the OptiTrack motion capture system. This validation process ensures that the proposed method performs as expected and provides reliable results in real-world applications.**

I. Derivation of LiDAR Incident Angle Based on Geometric Constraints

Ideally, the calibration board and the x-axis of the LiDAR frame should be perpendicular, but since this cannot always be guaranteed, we employ the incident angle α of the SP LiDAR to compensate for this as shown in Fig. 1. We derive α from geometric relationships using D, L, and θ. D is the change in the measured distance to the board, calculated as $D = d_{\text{end}} - d_0$, where d_{end} and d_0 are the final and initial distances at which the LiDAR detects the board, respectively. L is the known grid and hole size, and θ is the rotation angle of the board, determined by tracking the MW frame of the calibration board in the image. For details on MW tracking accuracy and methodology, please refer to Sec. II.

We define β as the angle between the x-axis of the LiDAR frame and the board at $t = end$, as shown in Fig. 1. Since $\alpha = 90° - \beta - \theta$, knowing β allows us to determine α. To relate θ and β, we first compute d_y, the distance from the initial LiDAR measurement point (A in Fig. 1) to the rotation axis. Using the law of cosines, d_y is given by:

$$d_y^2 = L^2 + D^2 - 2LD\cos\beta. \tag{1}$$

By applying the law of sines, we derive the following equation that directly relates θ and β:

$$\frac{D}{\sin\theta} = \frac{\sqrt{L^2 + D^2 - 2LD\cos\beta}}{\sin\beta}. \tag{2}$$

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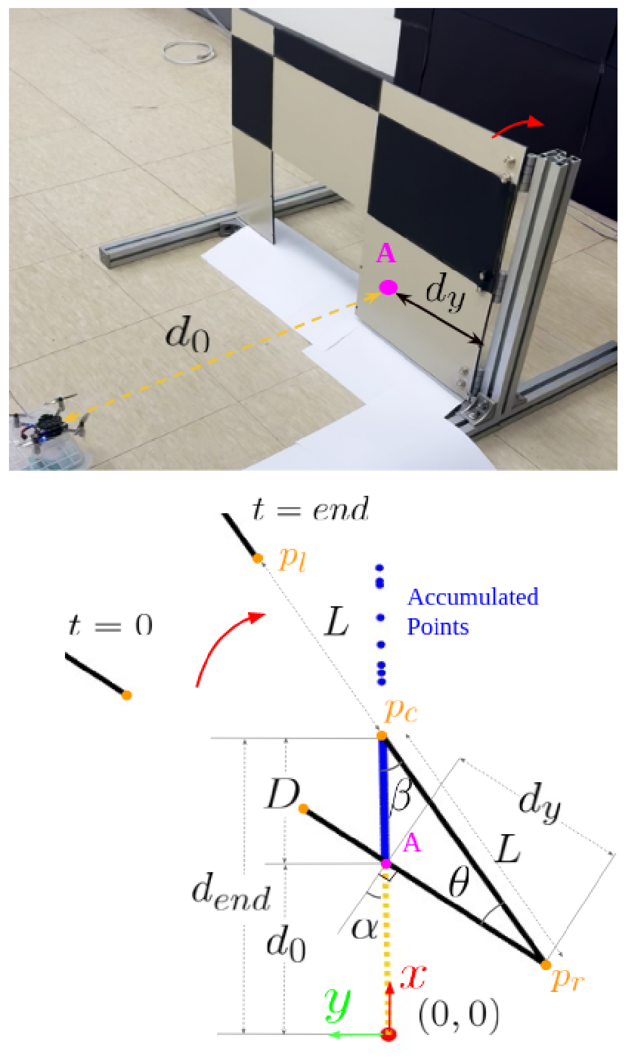

Fig. 1. The LiDAR measuring the calibration board at $t = 0$ (top) . Black lines (bottom) show the top view of the calibration board with a central hole, with symbol definitions detailed in Table I.

Solving for θ, we obtain:

$$\theta = \arcsin\frac{D\sin\beta}{\sqrt{L^2 + D^2 - 2LD\cos\beta}}. \tag{3}$$

The LiDAR must first detect the grid of the calibration board, not the hole. Therefore, β reaches its maximum when the LiDAR measures the edge of the board grid, as shown in Fig. 2 (left), where the value of β is $90° - \frac{\theta}{2}$. With θ known,

TABLE I

DEFINITION OF KEY SYMBOLS

Symbol	Definition
d_0	Distance when LiDAR first hits the calibration board
d_{end}	Distance when LiDAR last hits the calibration board
d_y	Distance between points A and p_r
θ	Rotation angle of the calib. board from $t = 0$ to end
α	Incidence angle of LiDAR to the calib. board at $t = 0$
L	Width of the square grid and hole

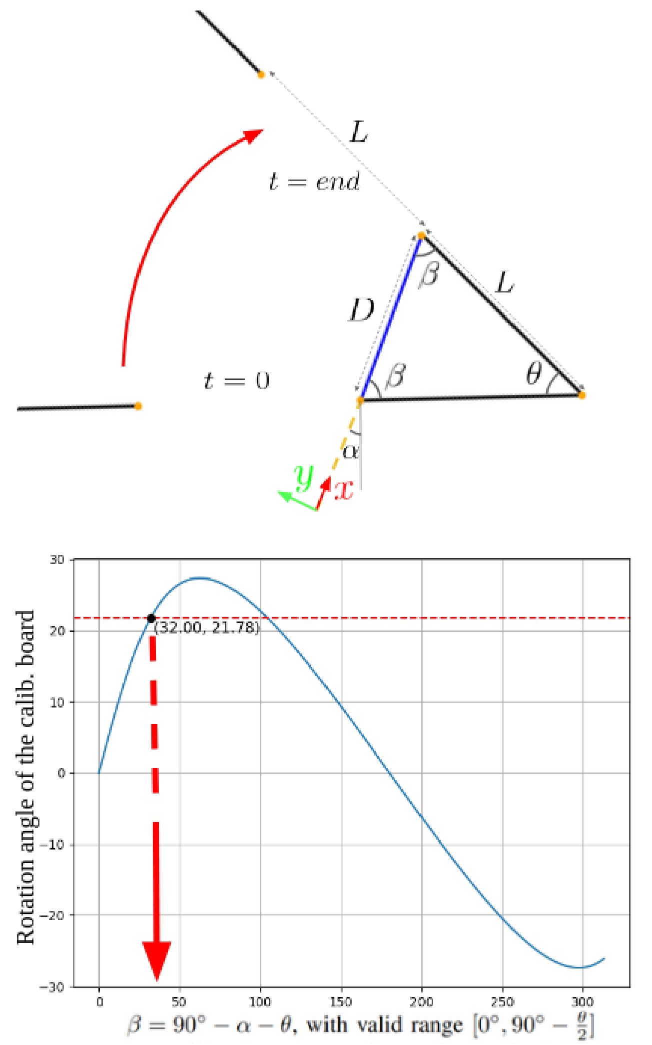

Fig. 2. When β is at its maximum (right), The relationship between θ and β follows Eq. 3 (right). Using the rotation angle θ of the board obtained from Sec. II, β is determined.

we search for the corresponding β within the valid range of $[0°, 90° - \frac{\theta}{2}]$, as shown in Fig. 2 (right).

II. ACCURACY OF MW TRACKING FOR BOARD ROTATION ANGLE

We determine the rotation angle θ of the calibration board by tracking the Manhattan world (MW) frames from (a) to (b), as shown in Fig. 3. We validated the reliability of θ by comparing the accuracy of the MW frame tracking with the ground truth from the OptiTrack motion capture system.

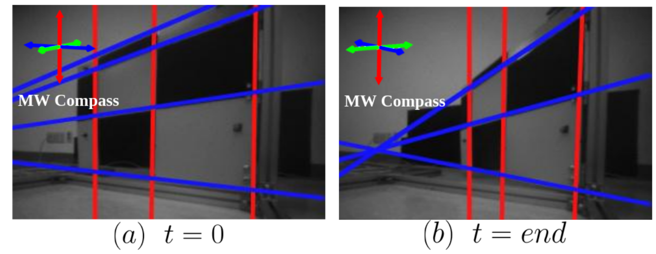

Fig. 3. Detection of the horizontal (blue) and vertical (red) lines on the calibration board, with the MW frame tracking results using these lines shown at $t = 0$ and $t = end$.

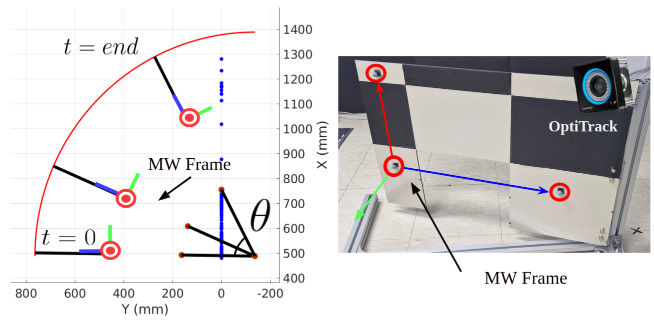

Fig. 4. (left) MW frame at each frame and (right) Ground truth MW frame created using markers.

As shown in Fig. 4 (left), we track the MW frame for each frame. We attach markers to the calibration board to obtain the ground truth of the MW frame, as shown in Fig. 4 (right). By comparing these two, the accuracy of the tracked MW frame is evaluated.

Table II compares the rotation angles between the MW frame at $t = 0$ and the six image frames, presenting the results for each frame individually. Our MW frame tracking is highly accurate, with θ tracked to an accuracy of 1° to less than 2°.

TABLE II

COMPARISON OF THE ABSOLUTE ROTATION ERROR [DEG] FOR θ

OptiTrack	11.35	21.56	41.61	55.62	73.29	90.02
MW Tracking	9.73	23.09	39.77	57.37	71.69	91.81
ARE	1.62	1.53	1.84	1.75	1.60	1.79